1. The time dependent equation for radial diffusion from a capillary is

|  |  |  |
| --- | --- | --- |
| **Prob #** | **Points** | **Max** |
| **1a** |  | **20** |
| **1b** |  | **10** |
| **2a** |  | **15** |
| **2b** |  | **20** |
| **2c** |  | **15** |
| **2d** |  | **10** |
| **3** |  | **10** |
| **Total** |  | **100** |

1. Use separation of variables, with to obtain two ordinary differential equations, one for and one for . (Use as the constant.)

**Answer:**

Because is not a function of and is not a function of .

Divide by

The left hand side is a function of only, and the right hand side is a function of only, so they must be equal to the same constant.

1. Deduce the solution to one of the resulting equations. (Hint: You have seen the solution to one of the equations, but may not yet know the solution to the other.)

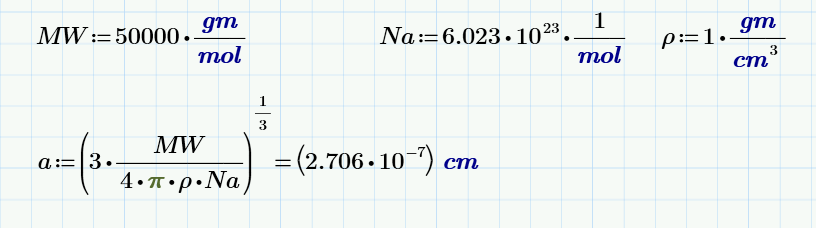
**Answer:**

For the time dependence, the solution is

1. A hollow fiber has a diameter of 200 m and a length of 10 cm. The nominal molecular weight cutoff for the pores (NMWCO) is 50,000 g/mole.
2. What is the pore radius in cm?

**Answer:**

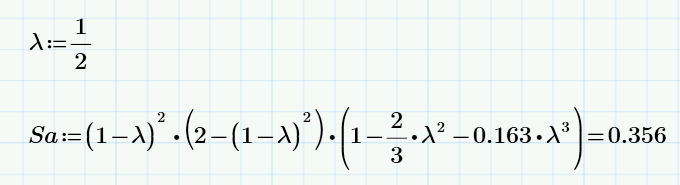
We can approximate the pore radius as the molecular radius of the largest molecule that will pass through the walls.



1. Find the actual sieving coefficient of a solute of molecular radius of cm for a fiber of this length and radius, but with a pore radius of cm.

**Answer:**

where

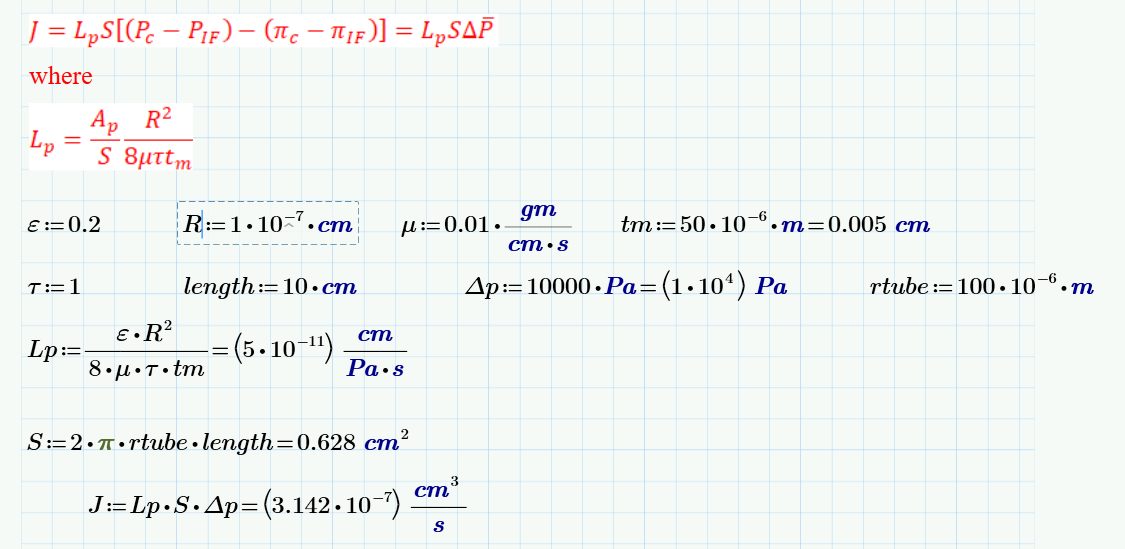


1. For the same fiber in Part b, find the total plasma flux, , assuming that the osmotic pressure is negligible and that the mechanical pressure across the wall is 10,000 Pa. Assume that , , pore tortuosity is 1, and the hollow fiber wall thickness is 50 m.

**Answer:**

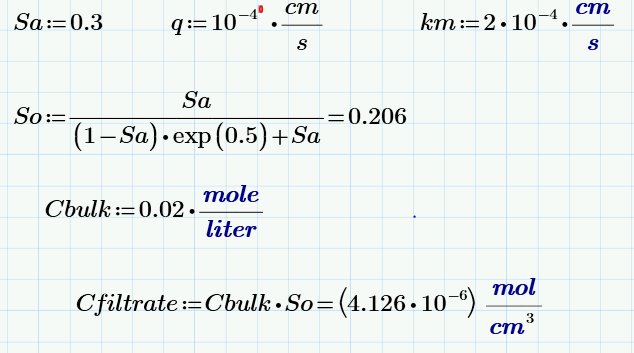
where

MathCad gives



1. Find the filtrate concentration of a hypothetical molecule in this fiber if the actual sieving coefficient is 0.3, the solvent flux per unit area is cm/s, the mass transfer coefficient is cm/s. Assume that the concentration of this drug is 0.02 M in the bulk solution.

**Answer:**



1. For oxygen diffusion into a spherical tissue, determine what condition must be met such that the oxygen concentration goes to zero at exactly the center of the sphere and the depletion of oxygen as a result of diffusion within the sphere is equal to the depletion of oxygen as a result of the mass transfer coefficient.

**Answer:**

The equation for the sphere is

At the sphere center it is

For this to be zero at the center of the sphere, we need

We can also write this in terms of the Damkholer number

If the effect of diffusion within the sphere is the same as the effect of the mass transfer coefficient

Or

**Potentially Useful Formulas**

# Solute Flux

# Nernst Equation

# Error Function

# Differential Equations

|  |  |
| --- | --- |
|  |  |
| Fourier Equation | or |
| Similar to 4, but with negative linear term | or |

**Fick’s First Law of Diffusion**

# Non-Dimensional Parameters

Let be a characteristic velocity, be a characteristic length, be kinematic viscosity,

Further, let be diffusion coefficient.

Let be the mass transfer coefficient.

# General Mass Transport Equation (Fick’s Second Law)

In Cardesian coordinates

In cylindrical coordinates

In spherical coordinages

# Boundary Layer Development

Pipe flow, fully developed momentum boundary layer .

Pipe flow, fully developed concentration boundary layer .

Flat plate, laminar boundary layer .

Cylinder, laminar flow .

# Constants

Avagadro’s Number:

Faraday’s Constant:

Universal Gas Constant:

Centigrade to Kelvin: Degrees Kelvin 273.15 Degrees Centigrade

**Table of Laplace Transforms**

|  |  |  |
| --- | --- | --- |
| 1 |  | 1 |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 23 |  |  |
| 24 |  |  |
| 25 |  |  |
| 26 |  |  |
| 27 |  |  |

# Sherwood Numbers

|  |  |
| --- | --- |
| Condition | Sh |
| Sphere in a stagnant fluid | 2 |
| Forced convection over a sphere |  |
| Laminar flow over a flat plate |  |
| Laminar flow in a cylindrical tube, short contact time |  |
| Laminar flow in a cylindrical tube, fully developed flow and concentration profiles | 3.66 |
| Turbulent flow through a circular tube |  |
| Spinning Disk |  |
| Falling Film, Average |  |

# Oxygen Concentrations

Henry’s Law

Sphere

Planar Bioartificial Organ

Perfusion Bioreactor